

Related Rates Cone Lab and Logger Pro

In this lab you will explore how the volume of a liquid in conical container changes as the container is filled. After collecting some data from a video clip, you will use your calculus knowledge to explore the rate of change of the volume of the liquid in the container.

Part I

1. *Opening the movie clip in Logger Pro.....*

Open the Logger Pro software. Under *Insert*, select *Movie*, and then navigate to the folder in which you saved your movie. The first frame of the video clip should appear in a window.

2. *Viewing the movie clip.....*

View the movie by pushing the play button on the left of the bar at the bottom of the movie window. Several frames may be played at the start before the water is poured in the glass. Rewind the movie by dragging the slider at the bottom of the window all the way back to the left. Also try using the forward and backward step buttons to view the video frame-by-frame. Note that on the top left of the movie window the frame number (or the frame time) is indicated for the frame that is currently displayed in the window.

Some questions to discuss with your group:

1. Think about how the height of the water is changing. Below, sketch a rough graph for height of water versus time.
2. Think about how the radius of liquid in the cone is changing. On the same set of axes below, sketch a graph for the radius of the liquid in the cone versus time. Label which is the radius and which is the height.
3. Consider the equation for the volume of a cone: $V = \frac{1}{3}\pi r^2 h$. Differentiate the volume function to determine an expression for $\frac{dV}{dt}$.
4. You can see that to find $\frac{dV}{dt}$, we need $\frac{dr}{dt}$ and $\frac{dh}{dt}$. You will now use LoggerPro to help you find values for these rates of change at various time values.

Part II

3. *Measuring position data from the movie clip.....*

You will now analyze the video clip by marking the positions of objects in each frame of the video.

- Drag the corner of the movie window to make the clip as large as possible for more accurate data collection.
- Click the button in the lower right corner with three red dots. This will open video analysis buttons.
- Click the button with the arrows to the left of the button with three red dots.
- Click **OK**. This will sync your data points with a graph.
- Now you will want to mark the points on the video. To do this, click the second button down on the right side (red dot with the crosshairs).
- As the liquid is being poured into the glass, you will mark points at the top of the liquid in the glass, **along the right edge of the glass**. Advance the video and then mark your points as the water rises. Mark between 7 and 10 data points. You may want to wait until the liquid “settles down” in the glass before marking your points.

Note: To delete any marked points, click on the top button on the right side (white arrow) to select the point and then press the delete key.

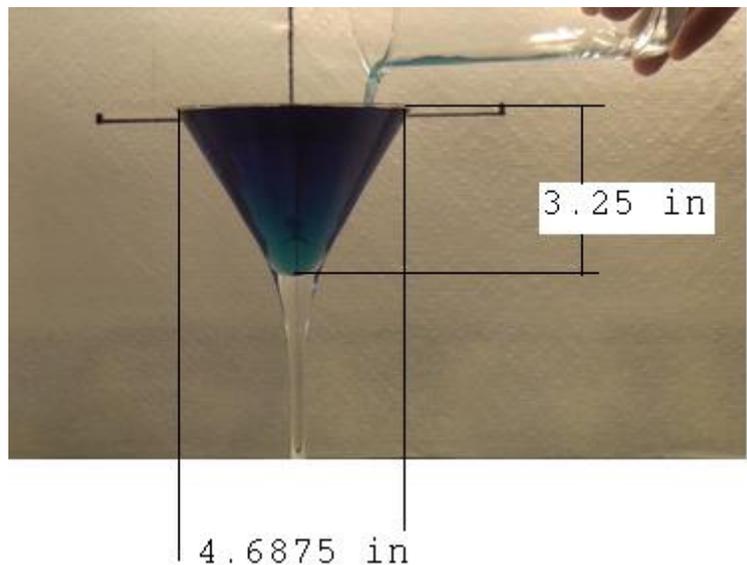
4. *Marking your origin and scaling the position data.....*

- Click the third button down on the right side with the graph to **set an origin**. You should set the origin at the bottom vertex of the cone.

Notice that the x- and y-coordinates are in pixels, not inches. You therefore need to create a scale for your video.

Note: The height of the conical portion of the glass measure 3.25 inches and the diameter is 4.6875 inches. As shown to the right.

Scale your graph by clicking the **horizontal ruler button** on the right panel. Click and hold the mouse at the first point you measure from, then while holding the mouse down, drag to the other point and release. You should see a green line appear on the video screen as you mark your scale. A box will then come up allowing you to set the actual distance.



5. Viewing the data table for the clip.....

You may notice that the points are being plotted on the graph behind the video.

Drag the right edge of the movie window towards the left, so the graph is visible. The data table is shown on the left of your screen. Click on that data table and scroll to the left. You should see time values in the first column (in seconds), and x-, and y-coordinates of the marked points in the second and third columns, respectively. The table also contains dx/dt and dy/dt at each t value. The values shown in red correspond to the red points on the graph, and the values shown in blue correspond to the blue data points on the graph.

You may want to “trace” the points on the LoggerPro graph. To do so, choose the Examine feature on the toolbar shown below. Then move your cursor on the data graph to see the specific coordinates associated with a data point.



Now that you have collected some data using the video clip and LoggerPro, you will use this data and some calculus to explore how fast the **volume** of the water in the cone is changing at various times.

Using the data collected in Logger Pro, answer the following questions.

1. What do ‘x’ and ‘y’ represent in the context of this problem?
2. Take a look at the graphs for the height and radius of the water in the cone produced by Logger Pro. On the same set of axes graph height versus time and radius versus time. Include a scale and units. Compare these graphs to the ones you sketched in Part II above and write a sentence to explain how these compare to the graphs you drew earlier.
3. By looking at the data in the Logger Pro file and choosing 4 time values for the collected data, fill in the table below: Include units.

Time	Radius	Height	$\frac{dr}{dt}$	$\frac{dh}{dt}$

4. Using the data in your table above and your calculus work from #1, find the value of $\frac{dV}{dt}$ for the time values you chose in #3 above. Show your steps in an organized fashion. Include units.

Time	$\frac{dV}{dt}$

5. The person pouring the liquid was trying to fill the container at a steady rate. Using the information in the table above, explain whether he was successful. That is explain how $\frac{dV}{dt}$ changes over the time span you considered.